# MJ Factorial 

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#### Abstract

In this era of the advancement of Physics, techonology and Computer science, as quantum mechanics and string theory is emerging, great demand of AI, fast search engines, there is an urgent need for the effective and efficient ways to calculate factorials. For replacing the time taking classical sequential multiplication for getting factorials by rapid processes, some interesting and unexpectedly general methods are used in this paper as consecutive subtraction of the power of consecutive numbers, its generalizations, using Pascal's triangle and much more. Even after so many decades of the advent of factorials into the Mathematical world, no one had ever tried to find the relations between the factorials from addition prespective and prime factors of the factorials. But no worries! Here in this paper, so many innovative ways for finding factorials are waiting for the people out there to be found!


Index Terms-Binomial coefficients, Exponents, Factorials, imaginary number, Integers, Pascal's triangle, Prime factorization, primorial, transcendental number.

## 1Introduction

T${ }^{\top}$ He Universe is complex. It's Quantum! Nothing is absolute. Everything has just some probability to happen. You can't really state about anything's certainty. There are always infinite possibilities of happenings. So, the probability of me getting the "field medal" is $1 / \infty$. Even the electron in its orbital is likely to be just $90 \%$ of the time.

Probability is a measure quantifying the likelihood that events will occur. For example if there are five persons for five different jobs, the ordered probability or Permutation is 120. For the first job, any of them can get it, then for each of the person there is four other persons who can get the second job and this goes on till the fifth job when we'll left with one person for each of the scenarios. We'll end up with something like this; $5 \times 4 \times 3 \times 2 \times 1$, what we call Factorials (It is actually $5!$ ). So, $n!=n$ $\times(n-1) \times(n-2) \times \ldots \times 1$.

But actually, Factorials are lot more than sequential multiplications of 1 to $n$. To get $n$ ! Integers, Rationals, Irrationals and even Complex numbers can be used. Basically, I'm declaring that all the numbers discovered by mankind till this date and surprisingly even transcendental numbers can be used to get $n$ ! using MJ Factorials calculation method.

## 2 MJ Factorial using Real number

### 2.1 MJ Factorial using Integers

Although the traditional way to calculate factorials is by using positive integers, but in MJ calculation method using integers, the whole set of integers can be used with the condition of being consecutive. To get $n$ ! using integers, raise the sum of a choosen integer and consecutive integers to the $n^{\text {th }}$ power and then subtract their values $n$ times repeatedly. For example to get 2!, according to MJ Factorials, first raise the consecutive integers to the power of 2 , than subtract their values. The obtained difference has to be subtracted one more time. And finally 2 ! will be there. Some of the examples of MJ Factorials
using Integers are shown below.
Subtracting each power (integers raised to the $n$ ) from its consecutive right placed value in all MJ factorials calculation (for both real and complex numbers).


### 2.2 MJFactorial using Rational number

Rational number can also be used in MJ factorials. For this, first to an arbitrarily chosen rational number add consecutive Integers and raise the sum to the $n$, then subtract their values $n$ times repeatedly as in the case of Integers. Example of 5 ! is shown below.


Taking an arbitrary rational number " $5 / 7$ " as an example in this MJ factorial calculation.

### 2.3 MJ Factorial using Irrational number

Factorials can also be obtained using Irrational numbers by doing the same procedure as did above with Rational num-
bers.


## 3 Transcendental numbers

Even transcendental numbers can be used to get Factorials, using MJ Factorials calculation method!


## 4 Complex numbers

Based on the concept of real numbers, a complex number is a number of the form $a+b i$, where $a$ and $b$ are real numbers and $i$ is an indeterminate satisfying $i^{2}=-1$. For getting $n!$ using complex number, same procedures will be followed as in the
case of real number by raising the sum of a choosen complex number and consecutive integers to the $n^{\text {th }}$ power and subtract their values $n$ times repeatedly. For simplicity the value of $\alpha$ has been taken 0 in the examples shown below.


$$
1!=1
$$



## $50!=1$

And one of the important features of MJ factorials is that it can perfectly explain why $0!=1^{[1]}$. According to the definition of MJ form of Factorials, raise the sum of the chosen real or complex number and consecutive integers to the 0 and then sub-
tract the values zero times which literally means no subtraction at all. For simplicity, take the example of a natural number 2 and add consecutive integers to it and it will give 0 !, as shown in the example below.


## 6 MJ FACTORIAL TRIANGLE

Amazingly because of MJ Factorials, $n$ ! can be predicted using ognize as Pascal's triangle).
'Staircase of Mount Meru'[2], (which you would probably rec-

| Factorial | Corresponding formulae |
| :---: | :---: |
| $0^{\text {th }}$ row | $1 a^{0}$ |
| $1^{\text {st }}$ row | $1 b_{2}{ }^{1}{ }^{1}-1 b_{1}{ }^{1}$ |
| $2^{\text {nd }}$ row | $1 c_{3}{ }^{2}-2 c_{2}{ }^{2}+1 c_{1}{ }^{2}$ |
| $3{ }^{\text {rd }}$ row | $1 d_{4}{ }^{3}-3 d_{3}{ }^{3}+3 d_{2}{ }^{3}-1 d_{1}{ }^{3}$ |
| $4^{\text {th }}$ row | $1 e_{5}{ }^{4}-4 e_{4}{ }^{4}+6 e_{3}{ }^{4}-4 e_{2}{ }^{4} \quad+1 e_{1}{ }^{4}$ |
| $5^{\text {th }}$ row | $1 f_{6}{ }^{5}-5 f_{5}{ }^{5}+10 f_{4}{ }^{5}-10 f_{3}{ }^{5}+5 f_{2}{ }^{5}-1 f_{1}{ }^{5}$ |
| $6^{\text {h }}$ row | $1 g_{7}{ }^{6}-6 g_{6}{ }^{6}+15 g_{5}{ }^{6}-20 g_{4}{ }^{6}+15 g_{3}{ }^{6}-6 g_{2}{ }^{6}+1 g_{1}{ }^{6}$ |
| $7^{\text {th }}$ row |  |
| $8^{\text {th }}$ row |  |
| $9^{\text {th }}$ row | $1 k_{10}{ }^{9}-9 k_{9}{ }^{9}+36 k_{8}{ }^{9}-84 k_{7}{ }^{9}+126 k_{6}{ }^{9}-126 k_{5}{ }^{9}+84 k_{4}{ }^{9}-36 k_{3}{ }^{9}+9 k_{2}{ }^{9}-1 k_{1}{ }^{9}$ |
| $10^{\text {th }}$ row | $1 l_{11} 1^{10}-10 l_{10}{ }^{10}+45 l_{9}{ }^{10}-120 l_{8}{ }^{10}+210 l_{7}{ }^{10}-252 l_{6}{ }^{10}+210 l_{5}{ }^{10}-120 l_{4}{ }^{10}+45 l_{3}{ }^{10}-10 l_{2}{ }^{10}+1 l_{1}{ }^{10}$ |
| It is still the same pile of number. I've just reformed it a little bit for making its use in Factorials and to be called as MJ Factorial triangle. |  |

In MJ Factorial triangle variables can be replaced by the sum of any real or complex number and consecutive integers, raise to the $n$ in $n^{\text {th }}$ row, but the value of numbers should decrease from left to right. And there is an alternate pattern of -+-+-+
... arithmetic sign in between the different terms of the formulae in MJ Factorial triangle as in the expansion of $(a-b)^{n}$. As in 3rd row of "MJ Factorials triangle"; $1 c_{4}{ }^{3}-3 c_{3}{ }^{3}+3 c_{2}{ }^{3}-1 c_{1}{ }^{3}, c$ can be any real or imaginary number but $c_{4}>c_{3}>c_{2}>c_{1}$ and they
all should be the sum of real or complex number and consecutive integers raise to the 3rd power. Let's take an example, let $c$ $0\}^{3}, c_{2}=\{1+(-1)\}^{3}$ and $c_{1}=\{1+(-2)\}^{3}$, (you can take any desired consecutive integers to the sum of a choosen real or imaginary number, as $3 / 2+5, \sqrt{n}+1, \sqrt[3]{k i}-1$ etc). Placing the values in place of variables in the 3rd row of MJ Factorials triangle and solving;

## 7 Generalising MJ Factorial calculations

For the generalized equation of $n$ ! using consecutive subtraction of the power of consecutive numbers, number of terms will be $2^{n}$, in which all the terms get repeated following the
$=1$ and now add consecutive integers to it and raise the sum to the $3^{\text {rd }}$ power. As $c_{4}=\{1+1\}^{3}$, then $c_{3}=\{1+$ $1 c_{4}{ }^{3}-3 c_{3}{ }^{3}+3 c_{2}{ }^{3}-1 c_{1}{ }^{3}$
$=1(2)^{3}-3(1)^{3}+3(0)^{3}-1(-1)^{3}$
$=1 \times 8-3 \times 1+3 \times 0-1 \times(-1)$
$=8-3+0-(-1)$
$3!=6$
binomial coefficient shown in Pascal's triangle, which is one of the reason to find the hidden factorials in Pascal's triangle.

| Number of terms | Generalizations |
| :--- | :--- |
| $2^{0}$ | $0!=k^{0}$ |
| $2^{1}$ | $1!=\left\{k^{1}-(k+(-1))^{1}\right\}$ |
| $2^{2}$ | $2!=\left[\left\{k^{2}-(k+(-1))^{2}\right\}-\left\{(k+(-1))^{2}-(k+(-2))^{2}\right\}\right]$ |
| $2^{3}$ | $3!=\left[\left\{k^{3}-(k+(-1))^{3}\right\}-\left\{(k+(-1))^{3}-(k+(-2))^{3}\right\}\right]-\left[\left\{(k+(-1))^{3}-(k+(-2))^{3}\right\}-\left\{(k+(-2))^{3}-(k+(-3))^{3}\right\}\right]$ |
| $2^{n}$ | $\left.n!=\left[\left\{k^{n}-(k+(-1))^{n}\right\}-\left\{(k+(-1))^{n}-(k+(-2))^{n}\right\}\right]-\left[\ldots(k+(-n))^{n}\right\}\right]$ |

$10!=2^{8} \times 3^{4} \times 5^{2} \times 7$

## 8 Proof: both Real and Complex number can be USED IN MJ FACTORIAL CALCULATION TO FIND FACTORIALS.

Let's take the example of 2 !
$2!=1 b_{3}^{2}-2 b_{2}^{2}+1 b_{1}^{2}$
$\left\{\right.$ where $b_{3}>b_{2}>b_{1}$ and all are the sum of consecutive integers and real or complex number which means, $b_{2}=\left(b_{1}+1\right)$ and $b_{3}$ $\left.=\left(b_{2}+1\right)\right\}$

Now, placing all the values, we'll get;
$=\left\{\left(b_{1}+2\right)^{2}-2\left(b_{1}+1\right)^{2}+b_{1}^{2}\right\}$
$=\left\{b_{1}{ }^{2}+4 b_{1}+4-2 b_{1}{ }^{2}-4 b_{1}-2+b_{1}{ }^{2}\right\}$
$=2$

After cancelling out everything we are left with 2, this shows that whatever be the value of $b$ either real or complex number we'll always have Factorial in the end.

## 9 Prime ( $p$ ) Factorization of Factorials

According to the Fundamental theorem of Arithmetic, every integer greater than 1 either is a prime number itself or can be represented as a product of prime numbers ${ }^{[3]}$. Writing a number as a product of prime numbers is called prime factorization. Some of the Factorials' prime factorization (except 0 and 1 factorials) are:
$2!=2$
$3!=2 \times 3$
$4!=2^{3} \times 3$
$5!=2^{3} \times 3 \times 5$
$6!=2^{4} \times 3^{2} \times 5$
$7!=2^{4} \times 3^{2} \times 5 \times 7$
$8!=2^{7} \times 3^{2} \times 5 \times 7$
$9!=2^{7} \times 3^{4} \times 5 \times 7$

$$
\begin{aligned}
& 11!=2^{8} \times 3^{4} \times 5^{2} \times 7 \times 11 \\
& 12!=2^{10} \times 3^{5} \times 5^{2} \times 7 \times 11 \\
& 13!=2^{10} \times 3^{5} \times 5^{2} \times 7 \times 11 \times 13 \\
& 14!=2^{11} \times 35 \times 5^{2} \times 7^{2} \times 11 \times 13 \\
& 15!=2^{11} \times 3^{6} \times 5^{6} \times 7^{2} \times 11 \times 13 \\
& 16!=2^{15} \times 3^{6} \times 5^{3} \times 7^{2} \times 11 \times 13 \\
& 17!=2^{15} \times 3^{6} \times 5^{3} \times 7^{2} \times 11 \times 13 \times 17 \\
& 18!=2^{16} \times 3^{8} \times 53 \times 7^{2} \times 11 \times 13 \times 17 \\
& 19!=2^{16} \times 3^{8} \times 5^{3} \times 7^{2} \times 11 \times 13 \times 17 \times 19 \\
& 20!=2^{18} \times 3^{8} \times 5^{4} \times 7^{2} \times 11 \times 13 \times 17 \times 19 \\
& 21!=2^{18} \times 3^{9} \times 5^{4} \times 7^{3} \times 11 \times 13 \times 17 \times 19 \\
& 22!=2^{19} \times 3^{9} \times 5^{4} \times 7^{3} \times 11^{2} \times 13 \times 17 \times 19 \\
& 23!=2^{19} \times 3^{9} \times 5^{4} \times 7^{3} \times 11^{2} \times 13 \times 17 \times 19 \times 23 \\
& 24!=2^{22} \times 3^{10} \times 5^{4} \times 7^{3} \times 11^{2} \times 13 \times 17 \times 19 \times 23 \\
& 25!=2^{22} \times 3^{10} \times 5^{6} \times 7^{3} \times 11^{2} \times 13 \times 17 \times 19 \times 23 \\
& 26!=2^{23} \times 3^{10} \times 5^{6} \times 7^{3} \times 11^{2} \times 13^{2} \times 17 \times 19 \times 23 \\
& 27!=2^{23} \times 3^{13} \times 5^{6} \times 7^{3} \times 11^{2} \times 13^{2} \times 17 \times 19 \times 23 \\
& 28!=2^{25} \times 3^{13} \times 5^{6} \times 7^{4} \times 11^{2} \times 13^{2} \times 17 \times 19 \times 23 \\
& 29!=2^{25} \times 3^{13} \times 5^{6} \times 7^{4} \times 11^{2} \times 13^{2} \times 17 \times 19 \times 23 \times 29 \\
& 30!=2^{26} \times 3^{14} \times 5^{7} \times 7^{4} \times 11^{2} \times 13^{2} \times 17 \times 19 \times 23 \times 29 \\
& 31!=2^{26} \times 3^{14} \times 5^{7} \times 7^{4} \times 11^{2} \times 13^{2} \times 17 \times 19 \times 23 \times 29 \times 31 \\
& 32!=2^{31} \times 3^{14} \times 5^{7} \times 7^{4} \times 11^{2} \times 13^{2} \times 17 \times 19 \times 23 \times 29 \times 31 \\
& 33!=231 \times 3^{15} \times 5^{7} \times 7^{4} \times 11^{3} \times 13^{2} \times 17 \times 19 \times 23 \times 29 \times 31 \\
& 34!=2^{32 \times 3^{15} \times 5^{7} \times 7^{4} \times 11^{3} \times 13^{2} \times 17^{2} \times 19 \times 23 \times 29 \times 31} \\
& 35!=2^{32} \times 3^{15} \times 5^{8} \times 7^{5} \times 11^{3} \times 13^{2} \times 17^{2} \times 19 \times 23 \times 29 \times 31 \\
& 36!=2^{34} \times 3^{17} \times 5^{8} \times 7^{5} \times 11^{3} \times 13^{2} \times 17^{2} \times 19 \times 23 \times 29 \times 31 \\
& 37!=2^{34} \times 3^{17} \times 5^{8} \times 7^{5} \times 11^{3} \times 13^{2} \times 17^{2} \times 19 \times 23 \times 29 \times 31 \times 37 \\
& 38!=2^{35} \times 31^{7} \times 5^{8} \times 7^{5} \times 11^{3} \times 13^{2} \times 17^{2} \times 192 \times 23 \times 29 \times 31 \times 37 \\
& 39!=2^{35} \times 3^{18} \times 5^{8} \times 7^{5} \times 11^{3} \times 13^{3} \times 17^{2} \times 19^{2} \times 23 \times 29 \times 31 \times 37 \\
& 40!=2^{38} \times 3^{18} \times 5^{9} \times 7^{5} \times 11^{3} \times 13^{3} \times 17^{2} \times 192 \times 23 \times 29 \times 31 \times 37
\end{aligned}
$$

$k^{x}$ is being added to $(\alpha \times k)!(x$ and $\alpha$ can be any natural number and $k$ is always prime number). If $n=k^{x}$ then its factorial will be; $(n-1)!\times k^{x}$ and all the factorials after it will be the product
of $k^{(x+\alpha)}$ for $(n+\alpha \times k)$ !, where $\alpha$ depends on how much steps it is far away from $n$ in terms of $k$, until the multiple of $k^{(x+1)!}$. And if a number is not equal to $\alpha \times k$ then the exponent of $k$ in
all these instructions one can predict $n!$. Let's take an example of 15 !. Write down all the prime numbers $\geq 15$ ( $15 \#$, ' $\#$ ' is pronounced as Primorial ${ }^{[4]}$ ); $2 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13$. Now think of a number close to $k^{x}$ (where $k=p$ ) for each listed prime number. Let's think of 2 , a number close to $15(<15<)$ which would be $2^{n}$ is both 16 and 8 . If you consider $8\left(2^{3}\right)$ then its factorial will be the product of $2^{\left(2^{\wedge} 0+2^{\wedge} 1+2^{\wedge} 2\right)}=2^{7}$ and 15 is 3 step away from 8 (in respect to 2 ). So, from 8 ! to 10 ! there will be the multiplication $2^{1}$ but from 10 ! to $12!2^{2}$ will join the product because $2^{2}$ is the multiplicative factor of $12\left(3 \times 2^{2}\right)$. Then from 12 ! to 14 ! there will be the multiplication of $2^{1}$ to the product of 12 !. Now, 15! will be the product of $2^{7} \times 2^{(1+2+1)}=2^{11}$. Another way of approaching $2^{n}$ for 15 ! is by finding relation with $16\left(2^{4}\right)$. One of the factor in the prime factorization of 16 ! will be $2^{\left(2^{\wedge} 0+\right.}$ $\left.2^{\wedge} 1+2^{\wedge 2}+2^{\wedge} 3\right)=2^{15}$. To get 16 ! we need to multiply 15 ! by 24 , so to get 15 ! back we've to divide 16! By $24\left(2^{(15-4)}=2^{11}\right)$. Doing the same procedure for all the above listed prime numbers we'll get $2^{11} \times 3^{6} \times 5^{3} \times 7^{2} \times 11 \times 13=1307674368000(15!)$.

## 10 Addition rules for Factorials

The recursive definition of factorials is:
$(n+1)!=n!\times(n+1)$
(and now distribute the RHS)
$(n+1)!=n!\times n+n!$
So this means, one can get $(n+1)$ ! using $n!$, but what one can do to find $(n+k)$ ! using $n$ ! (where $k$ is a positive integer). We know $(5-4)=1$ because they are consecutive, but $(5-2)=? 5$ and 2 are not consecutive integers but we know that they are distant by $x$ number of consecutive integers. So, we can solve this problem as

$$
\begin{aligned}
(5-2) & =(5-4)+(4-3)+(3-2) \\
& =1 \times 3 \\
& =3
\end{aligned}
$$

In case of factorials we know that $(n+1)!-n!=n!\times n$. We can use this fact to determine $(n+k)!-n$ ! using the same logic as I did above with positive integers.
$(5!-2!)=(5!-4!)+(4!-3!)+(3!-2!)$
$(4!=2!\times 3 \times 4$ and $3!=2!\times 3$, in respect to 2$)$
$=\left(2!\times 3 \times 4^{2}\right)+\left(2!\times 3^{2}\right)+(2!\times 2)$
$=2!\left(2+3^{2}+3 \times 4^{2}\right)$
The generalization of the difference of two factorials in terms of $n$;
$(n+k)!-n!=n!\left\{n+(n+1)^{2}+(n+1) \times(n+2)^{2}+(n+1) \times(n\right.$
$\left.+2) \times(n+3)^{2}+\ldots+(n+1) \times \ldots \times(n+k-2) \times(n+k-1)^{2}\right\}$
the prime factorization of it's factorials will remain the same as in the previous factorial, i.e., $\alpha \times k$. If $n=k^{x}$ then $n!$ will have $k^{\left(k^{\wedge} 0+k^{\wedge} 1+k^{\wedge} 2+\ldots+k^{\wedge} x-1\right)}$ as one of it's prime factors. With the help o

## 11 Conclusion

In the early $12^{\text {th }}$ century Factorials were used to count Permutations by Indian scholars ${ }^{[5]}$. But now, it has been found buried deep inside the root of many areas of mathematics as in combinations, permutations, algebra via the binomial coefficients, calculus, probability theory, number theory and much more. There is also a connection of hyper cube with factorials. But the computation of factorials is not much efficient. Surely, MJ factorial triangle, some rules for the power of prime numbers appear in the prime factorization of factorials, generalization of MJ factorials calculation method and addition rules for factorials which I've developed in this paper will also help to improve the efficiency of the computation of factorials. MJ Factorial triangle also shows the relation of factorials with Pascal's triangle. The technique that I developed in this manuscript to find factorials using Pascal's triangle is neither purely the binomial expansion nor polynomial expansion, it is something more interesting! The fact that we can use either real or complex numbers to find $n$ ! is showing relation of factorials with the whole set of Complex numbers and may be with the help of these we can even extend factorials. I'm still working on it and there is a lot more hidden in factorials.

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## Number of terms $=k$

With the help of the above equation any factorials of large value can be obtained using smaller factorials by summing up the above equation with the smaller factorial, respect to which the equation is arranged.

